MATH 1A − QUIZ 2 − SOLUTIONS

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(1) *(8 points)* Consider the following function:

$$f(x) = \cos^{-1}(e^x)$$

(a) *(2 points)* Find the domain of f

We want $e^x \in \text{Dom}\left(cos^{-1}(x)\right) = [-1, 1]$, hence $-1 \leq e^x \leq 1$.

However, $-1 \leq e^x \leq 1 \Rightarrow e^x \leq 1$ (because $e^x > 0$ for all x) $\Rightarrow x \leq 0$.

Hence: $\boxed{Dom(f) = (-\infty, 0]}$

(b) *(2 points)* Find the range of f

Notice that $0 < e^x \leq 1$ (because $x \leq 0$ here)

Hence $\cos^{-1}(0) > \cos^{-1}(e^x) \geq \cos^{-1}(1)$ (because $\cos^{-1}(x)$ is decreasing).

That is: $\frac{\pi}{2} > f(x) \geq 0$

Hence, we get: $\boxed{Ran(f) = [0, \frac{\pi}{2})}$

(c) *(2 points)* Show that f is one-to-one

Suppose $f(x) = f(y)$, then:

$$\begin{aligned}
\cos^{-1}(e^x) &= \cos^{-1}(e^y) \\
\cos\left(\cos^{-1}(e^x)\right) &= \cos\left(\cos^{-1}(e^y)\right) \\
e^x &= e^y \\
\ln(e^x) &= \ln(e^y) \\
x &= y
\end{aligned}$$

Hence $x = y$, and we're done!

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(d) *(2 points)* Find a formula for $f^{-1}(x)$

1) Let $y = \cos^{-1}(e^x)$

2) Then:

$$\begin{aligned}\cos^{-1}(e^x) &= y \\ \cos\left(\cos^{-1}(e^x)\right) &= \cos(y) \\ e^x &= \cos(y) \\ \ln(e^x) &= \ln(\cos(y)) \\ x &= \ln(\cos(y))\end{aligned}$$

3) Hence: $\boxed{f^{-1}(x) = \ln(\cos(x))}$

(2) *(2 points total)* Evaluate the following limits:

(a) *(1 point)*

$$\lim_{x \to 1^+} \ln\left(x^2 - 1\right) = \ln\left(1^+ - 1\right) = \ln(0^+) = -\infty$$

(b) *(1 point)*

$$\lim_{x \to \frac{1}{2}} \sin^{-1}(x) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

(because $\sin^{-1}\left(\frac{1}{2}\right)$ is the angle θ such that $\sin(\theta) = \frac{1}{2}$, that is $\theta = \frac{\pi}{6}$)